Pricing Services With Strong Externalities
A Representative Example

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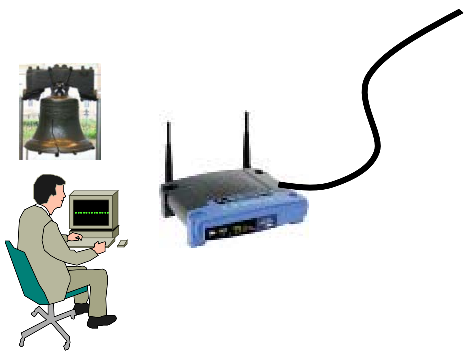
2012 Workshop on Internet Topology & Economics (WITE)

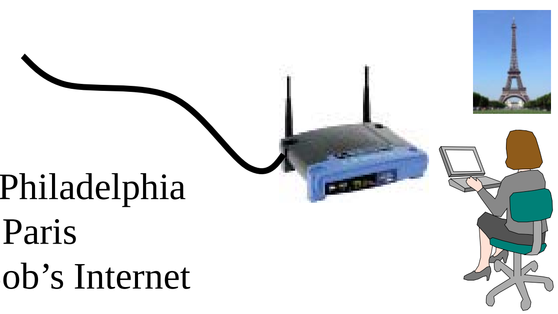
Background and Motivations

- We live in a networked world
  1) Connectivity as a basic service and a slew of other services that depend on it
  2) With connectivity come a wide range of new (positive & negative) interactions associated with services
- Network externalities – A service value depends in part on its users
  - Positive: More users $\Rightarrow$ Greater value (*e.g.*, Metcalf's law)
  - Negative: More users $\Rightarrow$ Lower value (*e.g.*, congestion)
- Given 1) and 2), we are interested in understanding
  - When and how will those new services succeed?
  - How should they be priced for success?

A Representative Example
User-Provided Connectivity

Internet





- Bob’s home connection to the Internet is in Philadelphia
- Jane’s home connection to the Internet is in Paris
- When Jane is in Philadelphia, she can use Bob’s Internet connection, and vice versa

- Users allow others to access their “home” connectivity in exchange for compensation/reciprocation
  - Cost sharing, payments, or reciprocated network access
  - Community-based networks as well as commercial offerings, *e.g.*, FON, OpenGarden, Keywifi, AnyFi
- Service exhibits strong externalities that shape its eventual success
  - Positive: More users ⇒ More connectivity options
  - Negative: More users ⇒ Greater odds of sharing connectivity and/or facing congestion

Modeling a UPC Service Offering

- Service adoption based on a *utility* function
  - Users make individual decisions and adopt only if they derive *positive* utility
  - Users are *heterogeneous, i.e.*, differ in how they value "roaming" Internet access
    - θ (roaming parameter) is a private random variable with known distribution

- Service utility for user with roaming characteristic θ is of the form

$$U(\theta) = F(\theta,x)+G(m) - p(\theta)$$

  - $F(\theta,x)$ is utility of connectivity (at home and away) and increases with x (*positive externality*)
  - $G(m)$ captures impact of traffic m from other users (*negative externality*), as well as possible compensation for providing Internet access for that traffic
    - Note m depends on both the number *and* type (their θ value) of adopters
  - $p(\theta)$ is price charged to user with characteristic θ

From Model to Tractable Model

- User utility : *Linear* (positive and negative) externalities

$$U(\theta) = (1- \theta)\gamma + r\theta x - cm - p(\theta)$$

  - θ : uniform in [0,1] – $\theta = 0$: sedentary; $\theta = 1$: always roaming
  - γ : value of home connectivity – weighed by frequency of use $(1 - \theta)$
  - r : value of roaming connectivity – weighed by likelihood (proportional to coverage x) and frequency of use θ
  - c : impact of roaming traffic m (either at home or while roaming)
    - m depends not just on number of adopters, but *who* they are (their θ values)
    - for simplicity, *m is* assumed uniformly distributed across home connections
  - $p(\theta)$: service price to user of type θ
- Provider's profit :
  - Per user profit : $\pi(\theta) = p(\theta) - e$ (e, per user service deployment cost)
  - Total provider profit : $\prod = \int_{\{U(\theta)>0\}} \pi(\theta)\, d\theta$

Understanding Service Valuation

- How valuable is the service overall and to individual users?
  - Different users see a different utility $U(\theta)$
  - Provider may extract a different profit $\pi(\theta) = p(\theta) - e$ from each user
- Value $v(\theta,x) = U(\theta)+\pi(\theta)$ of user θ given adoption level x and $U(\theta)>0$
  - $v(\theta,x) = \overbrace{(1-\theta)\gamma + r\theta x - cm \underbrace{- p(\theta)}}\, \underbrace{+ \overbrace{p(\theta)} - e} = \gamma + \theta(rx - \gamma) - cm - e$
  - $p(\theta)$ as a value transfer "knob" between provider and users
- Overall service value $V(\Theta,x)$ for a set of adopters Θ, s.t. $|\Theta|=x$

$$V(\Theta,x) = \int_{\theta\in\Theta} v(\theta,x)\, d\theta$$

- Question(s) :
  - Given x, what is the maximum value of $V(\Theta,x)$ and what Θ^* realizes it?
  - What is the value x^* that maximizes $V(\Theta^*,x^*)$?

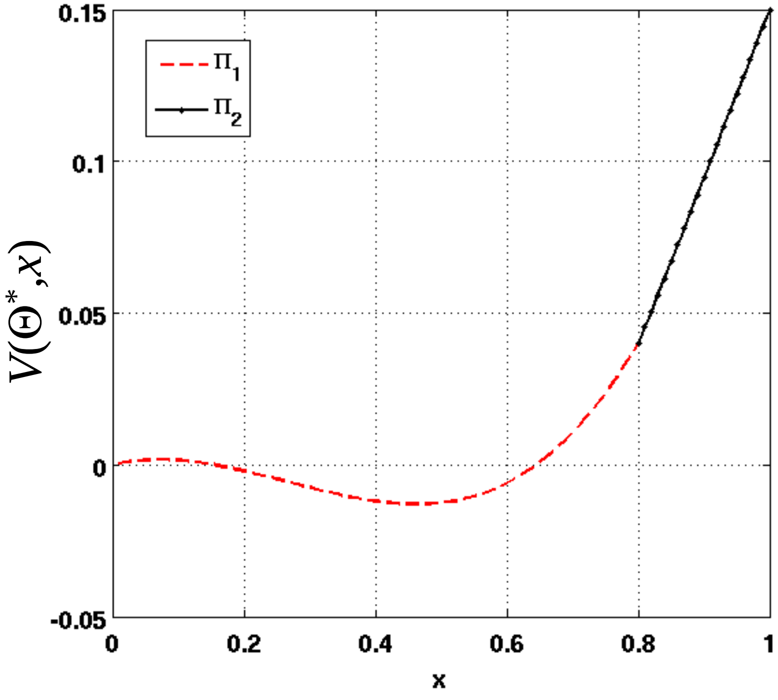
Maximizing Service Valuation

- Given an adoption level *x*, what Θ^* maximizes $V(\Theta,x)$?

$$\Theta^* = \begin{cases} [0,x) & \text{if } \; x < \dfrac{\gamma}{r-c} \\ [1-x,1] & \text{if } \; x \geq \dfrac{\gamma}{r-c} \end{cases}$$

- Optimal adoption is always for a *contiguous* set of users
  - Adoption *threshold* determines range of adopters
- Computing x^* can be done by optimizing *x* across both ranges, and selecting the value that yields the largest service valuation

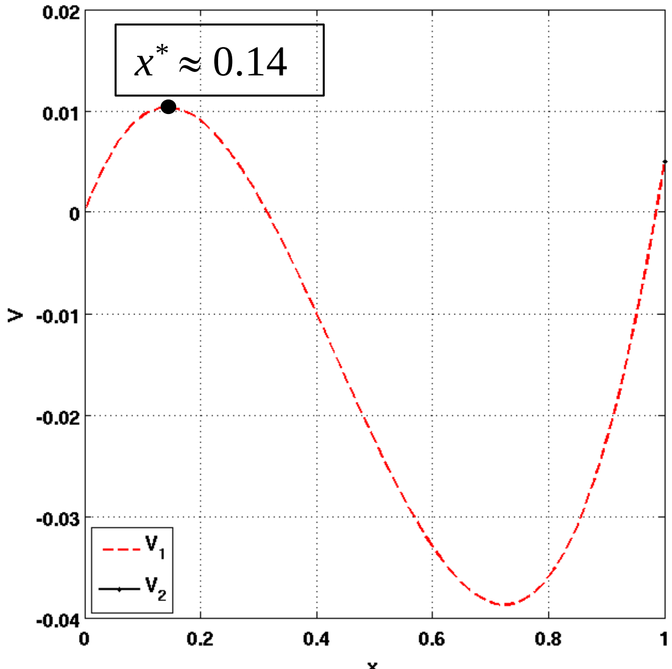
Note: In this example, $x^* = 1$



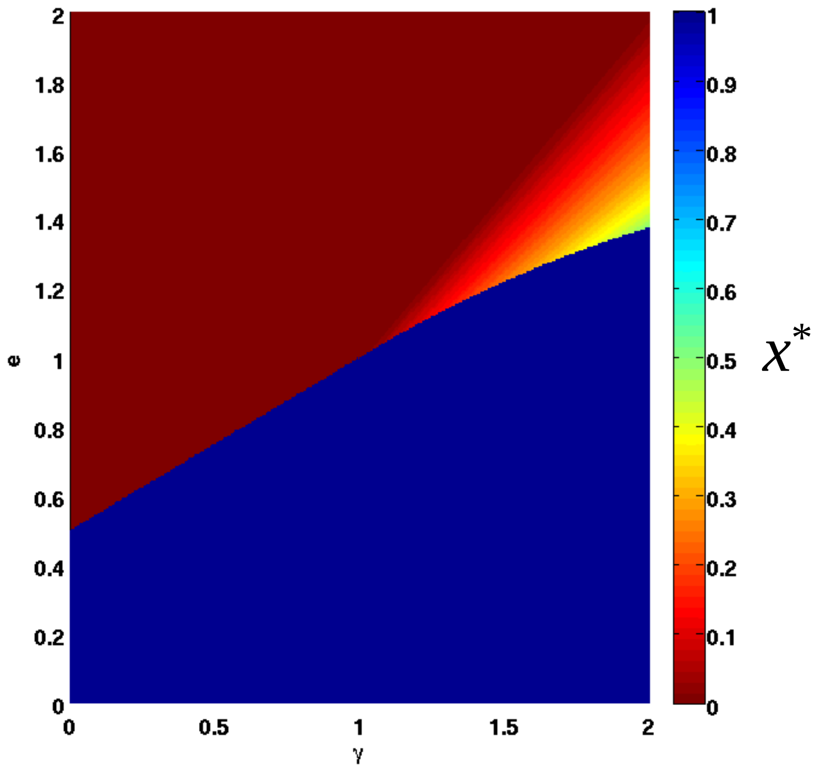
$\gamma = 0.8$, $e = 0.75$, $c = 0.6$, $r = 1.6$

When Is a UPC Service Valuable?

- In most cases, maximum value is realized at maximum adoption, but this need not always be so



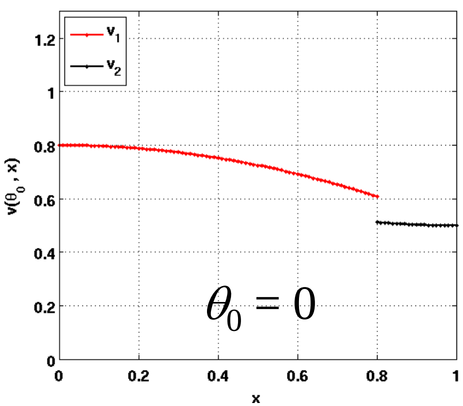
$\gamma = 1.3,\ e = 1.145,\ c = 0.6,\ r = 1.6$

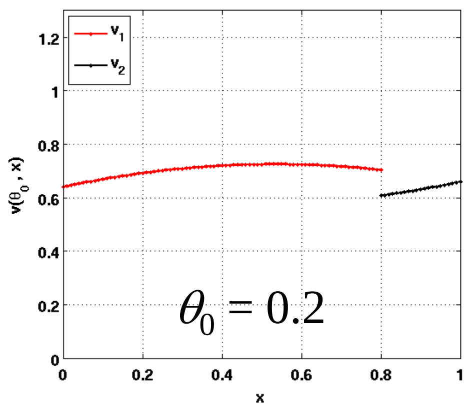


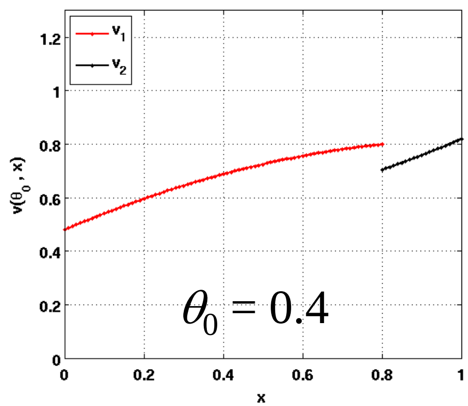
$r - c = 1$

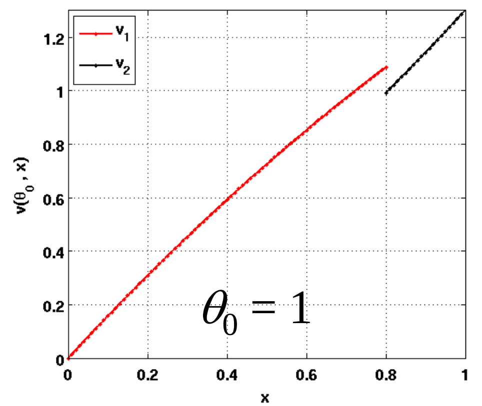
Variations in UPC Service Value

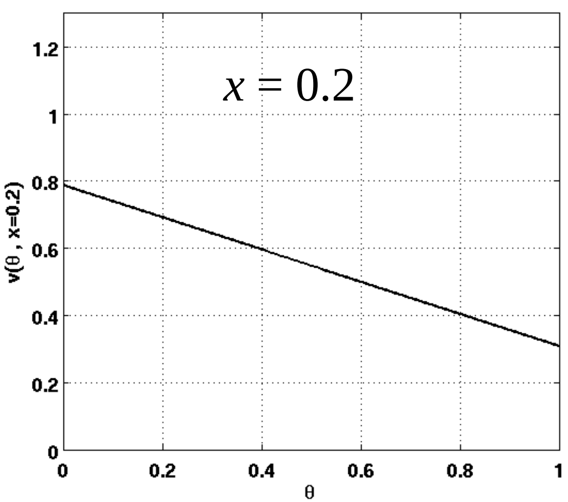
- Service value varies over both time and users ($\gamma = 0.8$, $e = 0$, $c = 0.6$, $r = 1.6$)

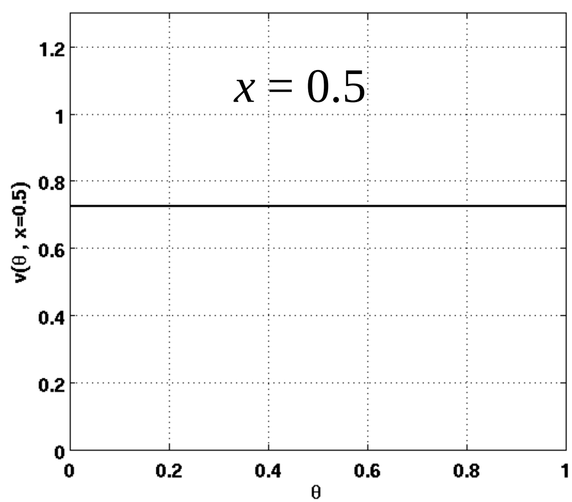


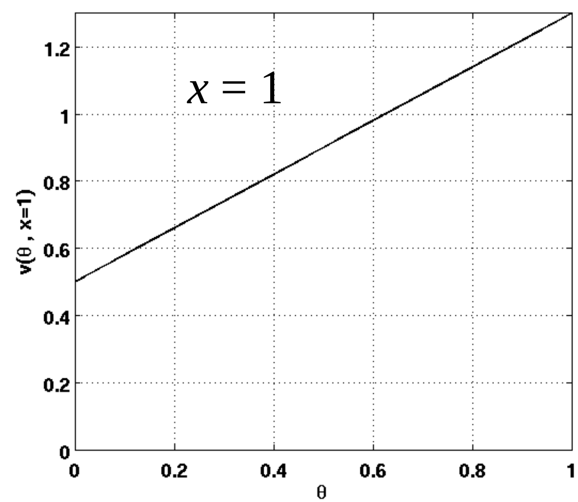












Realizing UPC Service Valuation

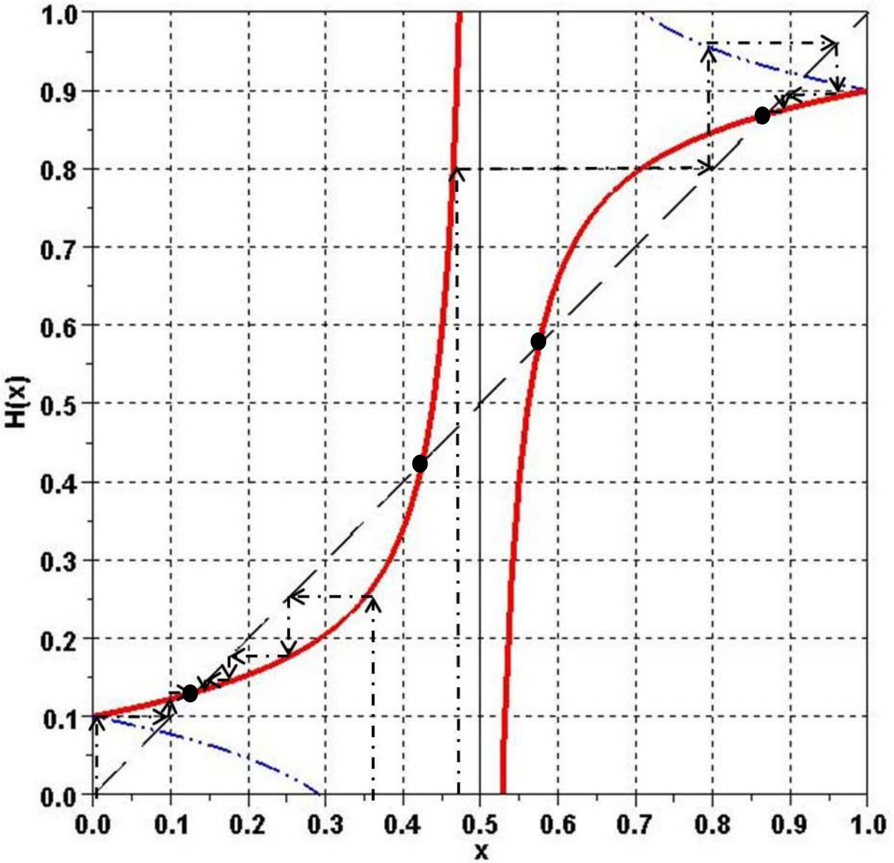
- When a UPC delivers substantial value (under high or full adoption)
  - Users' valuation is highly heterogeneous
    - High θ users see significantly higher value than low θ ones, but only once adoption is high enough
  - Users' valuation varies extensively as adoption level changes, and in different directions depending on a user's θ value
- Two key consequences
  1. Price discrimination is required to achieve full value
  2. Adoption dynamics can interfere with realizing system value

Service Realization Options

- Introductory price to “full” adoption, followed by discriminatory pricing to achieve desired adoption (cull unwanted users and realize full valuation)
  - Addresses adoption dynamics concerns, but still unrealistic due to discriminatory pricing assumption
- Two possible alternatives to full discriminatory pricing
  1. Single, time-varying price
     - Price varies over time, but all users pay the same price
     - Single price eliminates need for discriminatory pricing information, and time-varying nature addresses adoption dynamics, but single price across users wont capture valuation heterogeneity
  2. Multiple, time-dependent prices
     - Price varies over-time, but users keep their adoption price
     - Addresses adoption dynamics and leverages insight that higher valuation often arises as adoption increases

Adoption Dynamics Under the Single Price Option

- A simple discrete time model
  - Adoption *level* at epoch n+1, x_{n+1}, is determined by adoption *state* at epoch n, X_n (a two-dimensional quantity – number x_n ***and type*** y of adopters)
  - Users evaluate their utility based on X_n and adopt if it is non-negative, *i.e.*, $X_{n+1}=H(X_n)$
- Adoption evolves based on the shape and position of the function(s) $H(X)$ relative to X
  - Different functions before and after a transition to a state of high/low adoption
- Equilibria correspond to $H(X) = X$ (or $H(0) \leq \underline{0}$, or $H(1) \geq \underline{1}$)

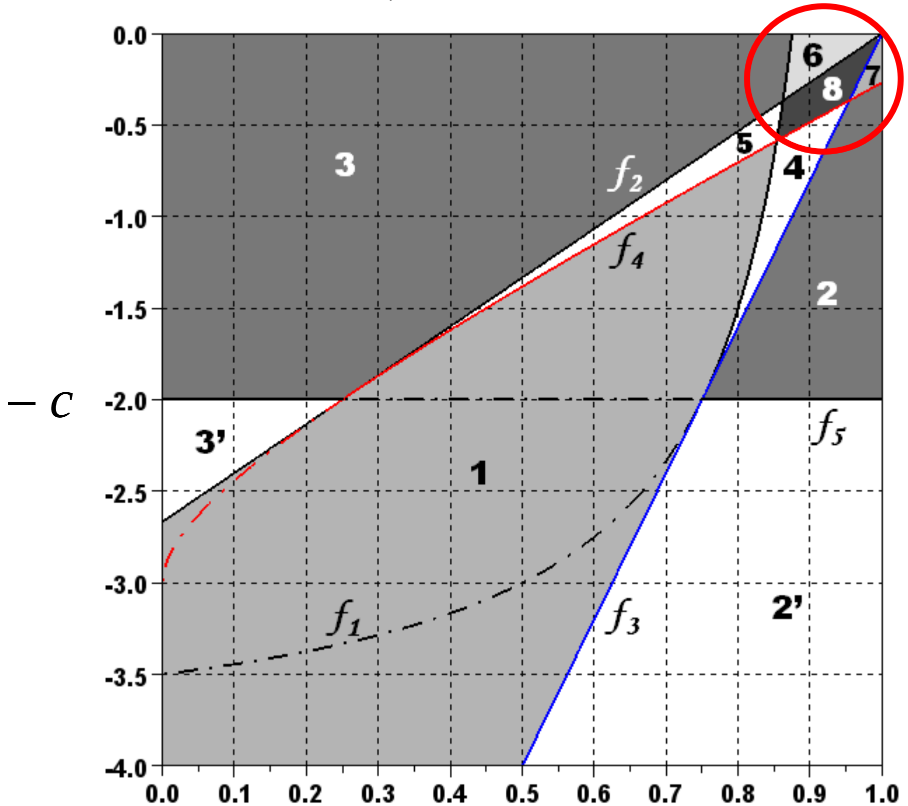


Single Price Adoption Outcomes

- Associated with different regions of the (p,c) plane
  - $U(\theta) = \gamma - cm + \theta(rx - \gamma) - p$
- Various possible combinations of equilibria or absence thereof

Cases	$[0, 1/2)$	$[1/2, 1]$
1	—	—
2	●	—
2’	↻	—
3	—	●
3’	—	↻
4	●, ○	—
5	—	●, ○
6	●, ○	●
7	●	●, ○
8	●, ○	●, ○

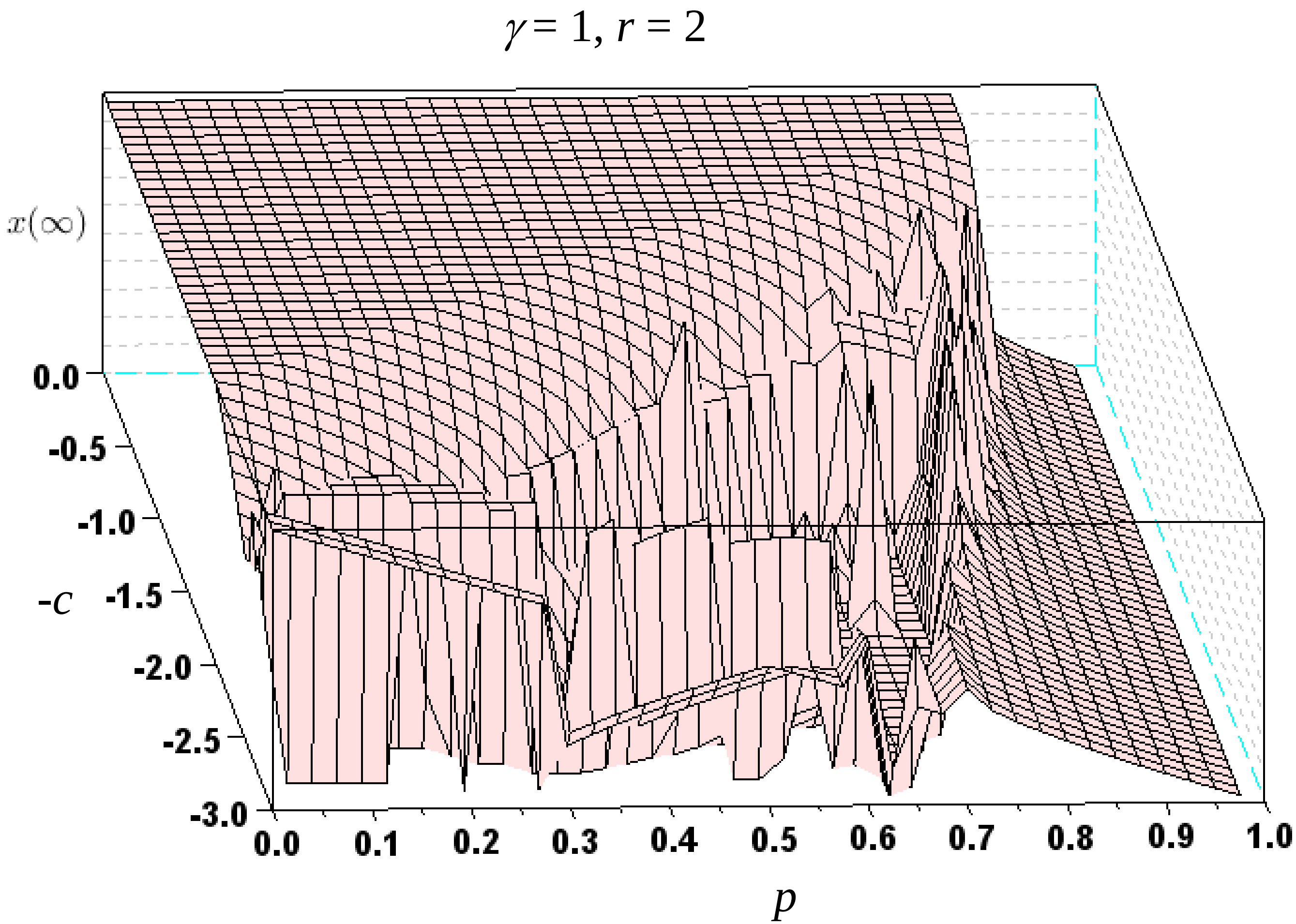
Case: $\gamma = 1$ and $r = 2$



p

Multiple equilibria can create challenging adoption dynamics

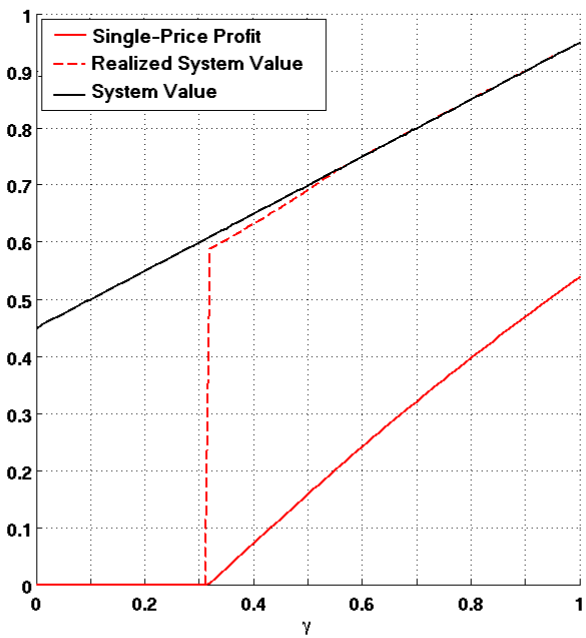
Adoption Equilibria



"Efficacy" of the Single Price Policy

- A significant gap between optimal profit and maximum value
  1. Inability to recover most value from high θ adopters
  2. Need to set initial price low enough to foster adoption and avoid regions 6, 7, 8
- Social wellfare can also be significantly affected, especially as *c* increases

$e = 0.3$, $c = 0.1$



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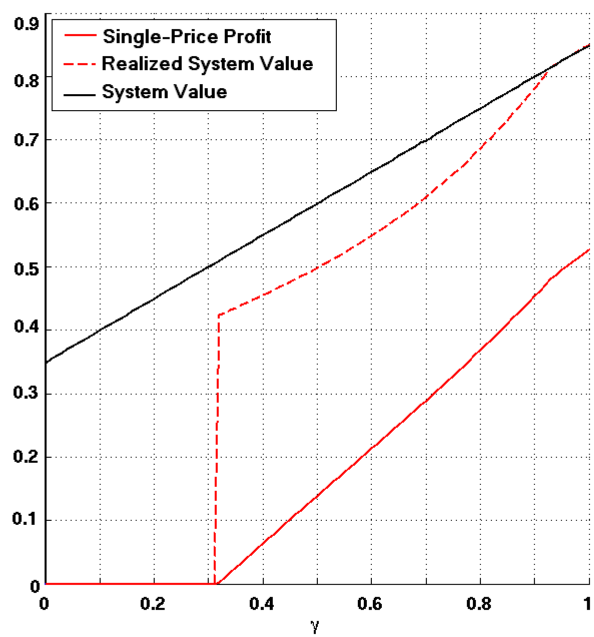
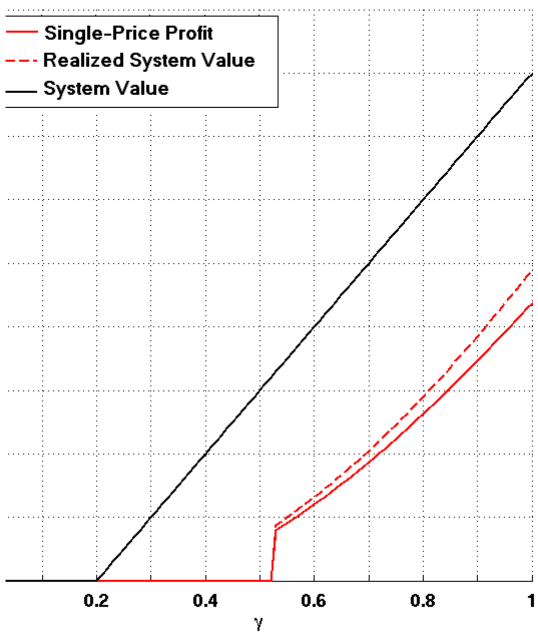


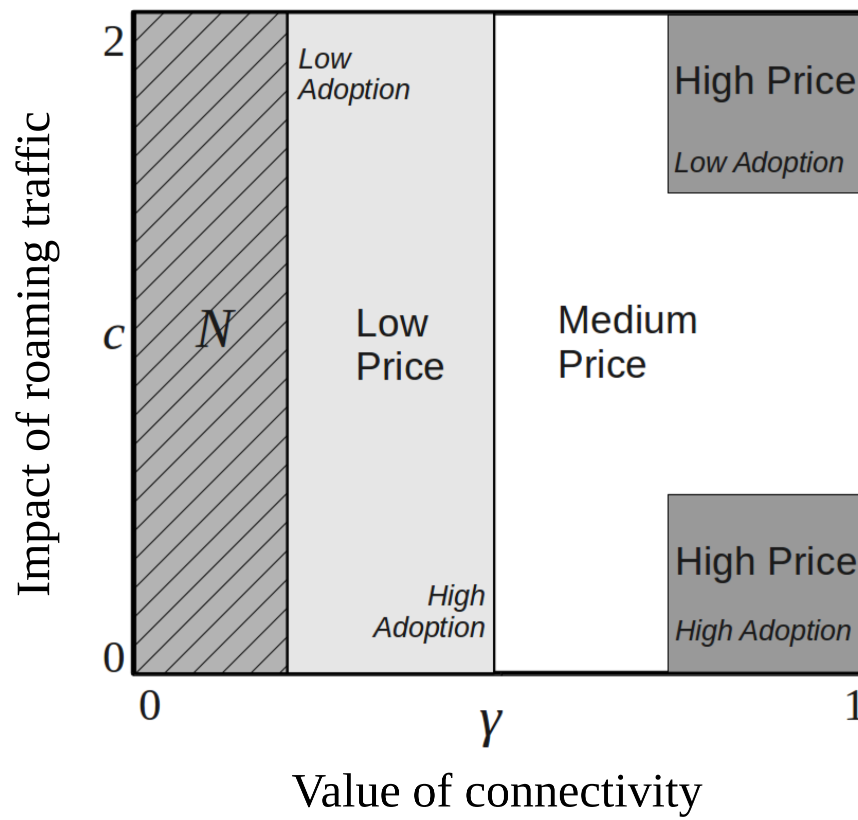
$e = 0.3$, $c = 1.2$



Guidelines for Single-Price Policy

- When connectivity utility is low, price must be set low
  - Low service adoption when negative impact of roaming traffic is high
  - High adoption otherwise
- When connectivity utility is high, a high price is optimal in two distinct scenarios
  - Roaming traffic has limited impact, and high adoption is feasible even when price is high
  - Roaming traffic has a major impact, and realizing high adoption would call for too low a price

$$p \in [e, \gamma + 1 - c/2]$$



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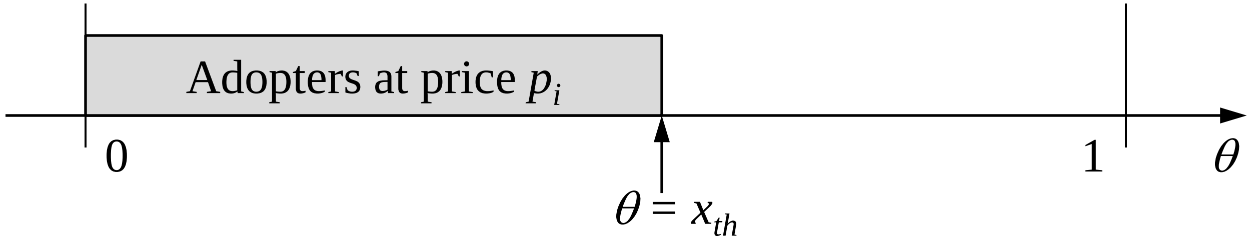
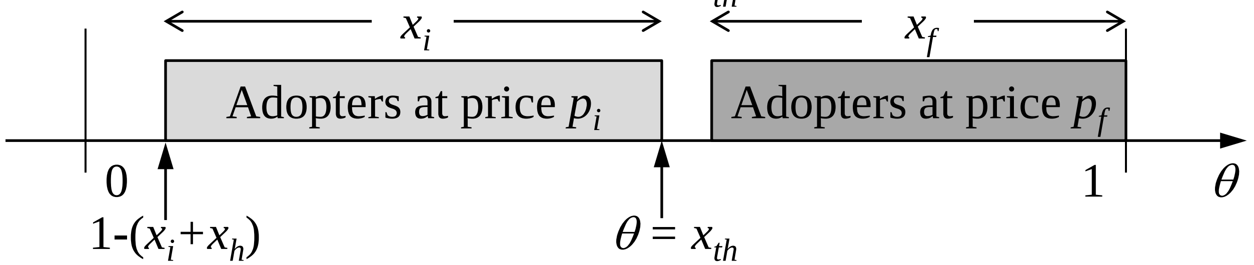
Two, Time-Dependent Prices Option

- Low initial price, p_i, builds-up adoption
- Higher final price, p_f, extracts added value (from high θ users) generated by high coverage
- Price switching based on adoption threshold x_{th}
- Profit: $\Pi^{(2)}(p_i,p_f,x_{th}) = (p_i - e)x_i + (p_f - e)x_f$
  - x_i and x_f are the fractions of adopters paying p_i and p_f
  - p_i^*, p_f^*, and x_{th}^* are selected so that $\Pi^{(2)}(p_i^*,p_f^*,x_{th}^*)$ is maximized

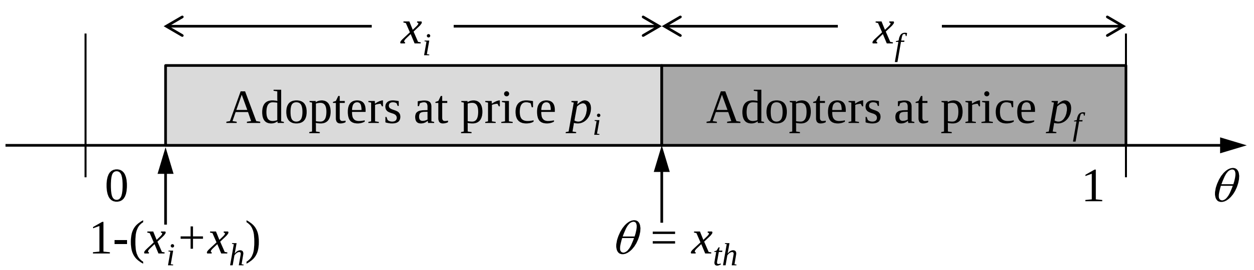
Adoption Under Two-Price Policy

- Complex (non-contiguous) adoption patterns can emerge





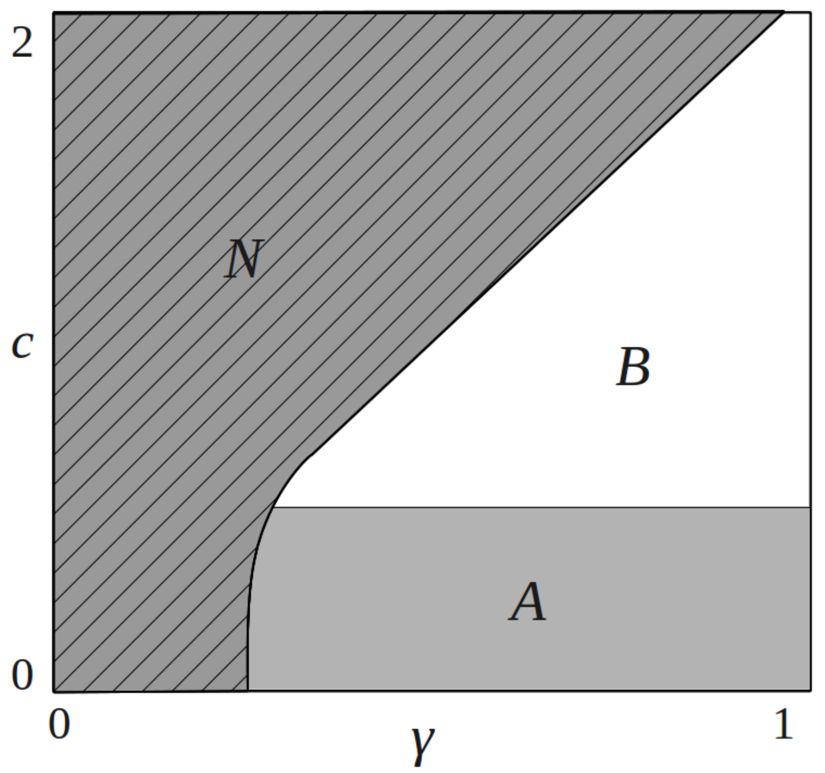
- For analytical tractability and in keeping with maximal value results, parameters are “constrained” to ensure contiguity of the adoption region



Guidelines for Two-Price Policy

Three distinct regions

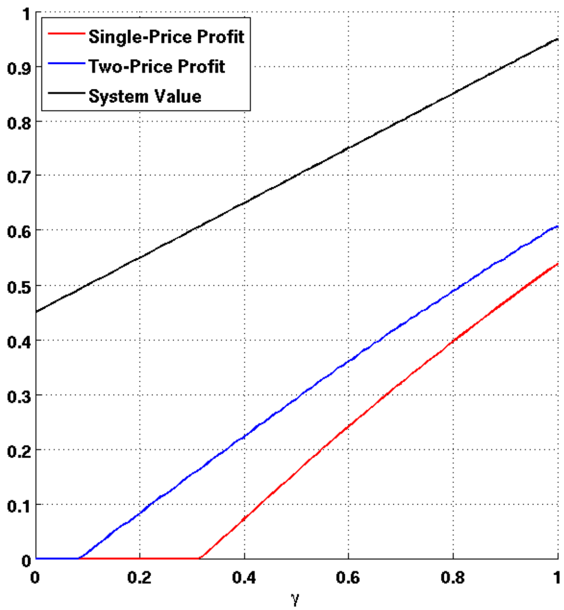
1. In region N (very low γ or γ that is too low relative to c), the two-price policy calls for subsidies ($p_i \approx 0$) that make it non-competitive
2. As we enter region B (γ increases or c decreases), both prices increase roughly linearly, while x_{th} stays fixed as some moderate value
3. In region A (low c), x_{th} switches to a higher value, prices are unaffected by further decreases in c, and increase linearly with γ



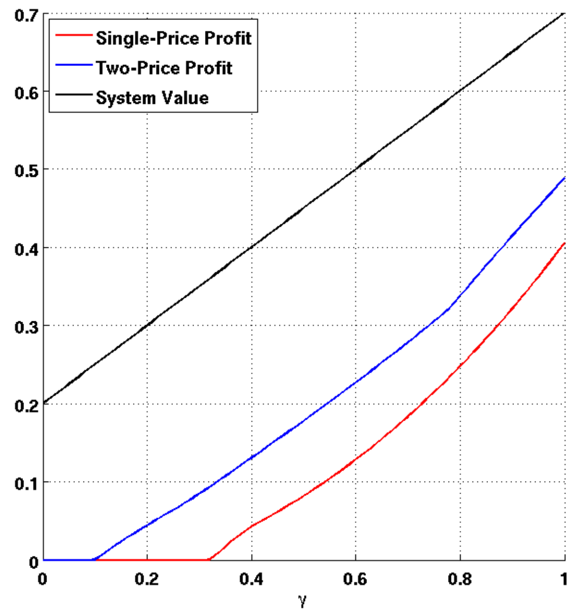
Effectiveness of the Two-Price Policy

- Some improvements over a single price policy, but still far from being able to extract the full service value

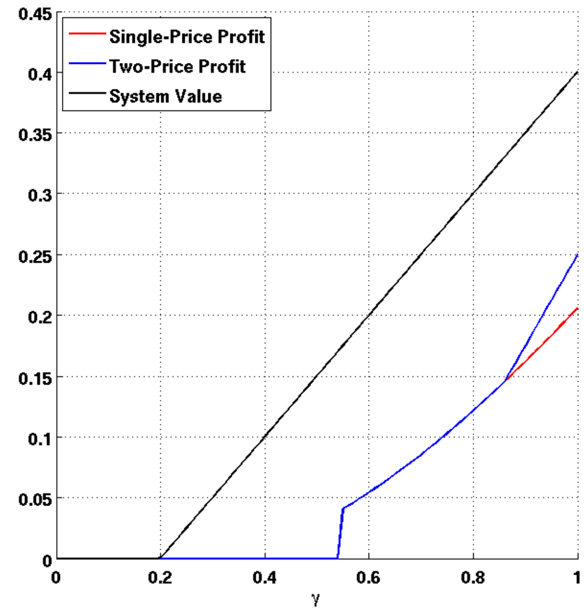
$e = 0.3$, $c = 0.1$



$e = 0.3$, $c = 0.6$

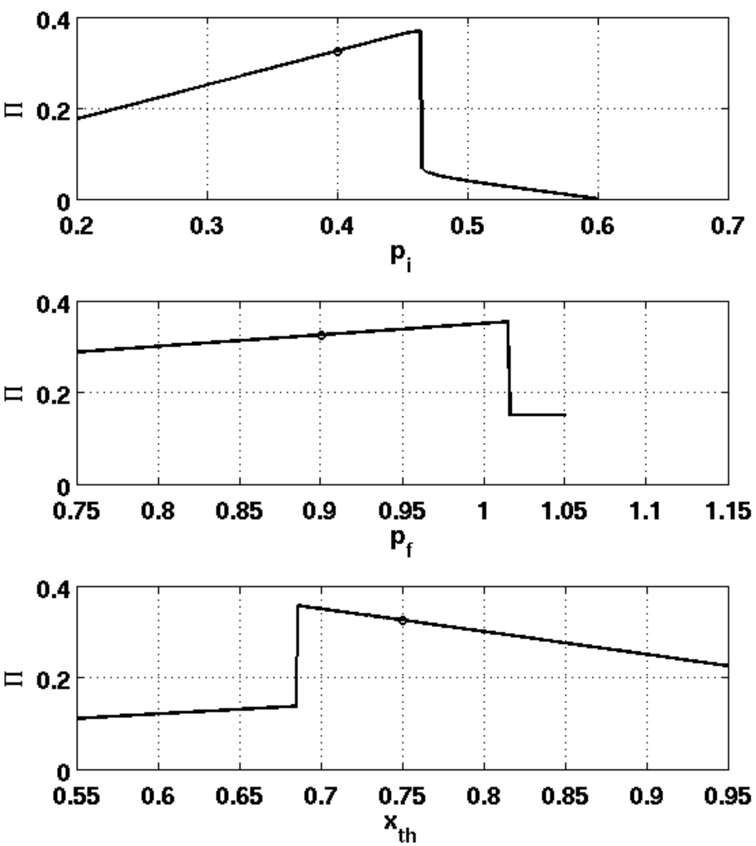


$e = 0.3$, $c = 1.2$



A Word of Caution

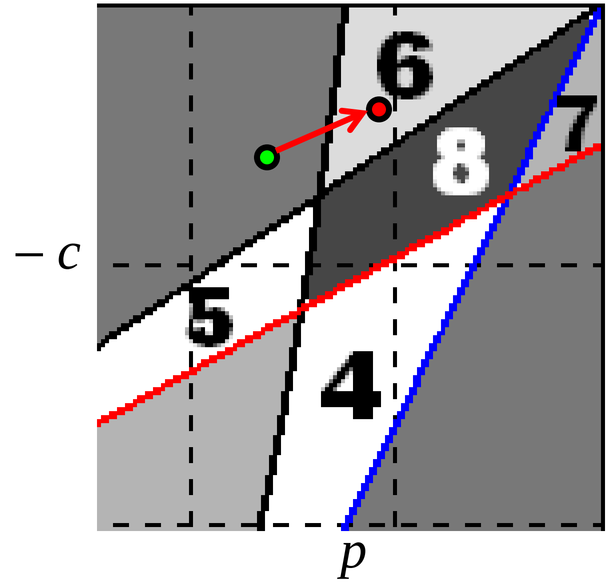
- Adoption levels experience sharp transitions after small changes in parameters around the "optimal" points
- This is *intrinsic* to the service adoption process
  - Similar behaviors are observed (numerically) when relaxing the model's assumptions, *i.e.*, non-linear externalities, different coverage and roaming distributions, etc.



On the Impact of Incentives

- Recall our user's utility function
  - $U(\theta) = (\gamma - p) - cm + \theta(rx - \gamma)$
- Incentives can be offered to offset the impact of roaming traffic, *i.e.*,
  - $U(\theta) = (\gamma - p) + (\boldsymbol{b} - c)m + \theta(rx - \gamma)$
- Incentives are equivalent to a price offset, *i.e.*, $p \to (p+bm) - bm$
- No impact on equilibria, but effect on adoption dynamics can be significant
  - Introducing incentives can lead to the creation of a ***second*** low adoption equilibrium

Increasing incentives from 0 to b, moves the operating point from $(-c, p_1)$ ● to $(\boldsymbol{b}-c, p_1+\boldsymbol{bm})$ ●



- In general, UPC adoption can be difficult to predict in the presence of multiple equilibria

Summary

- A UPC service involves both positive and negative externalities that depend on ***both*** the number and the type of users adopting it
  - This is not an uncommon situation with networked services
- A simple model was developed that offers
  - Insight into service valuation, *i.e.*, when and why is it valuable and for whom
- The model revealed that valuation varies significantly across users and as a function of adoption level
  - $\Rightarrow$ Dynamic and discriminatory pricing are needed to extract full value
- Full price discrimination is, however, often neither practical nor allowed
  - Single-price and two-price policies were investigated
  - A two-price policy improves profit, but still falls short of realizing maximum profit
- The model also identified
  - The fragility of "optimal" pricing strategies
  - The potentially ambiguous role of incentives in improving system adoption

References

[1]. M. H. Afrasiabi and R. Guerin, "*Exploring User-Provided Connectivity - A Simple Model.*" Proc. ICQT'11 Workshop, Paris, France, October 2011. (Tech. report version).

[2]. M. H. Afrasiabi and R. Guerin, "*Pricing Strategies for User-Provided Connectivity Services.*" Proc. IEEE INFOCOM 2012 mini-conference, Orlando, FL, March 2012. (Tech. report version).