Exploring User-Provided Connectivity
A Simple Model

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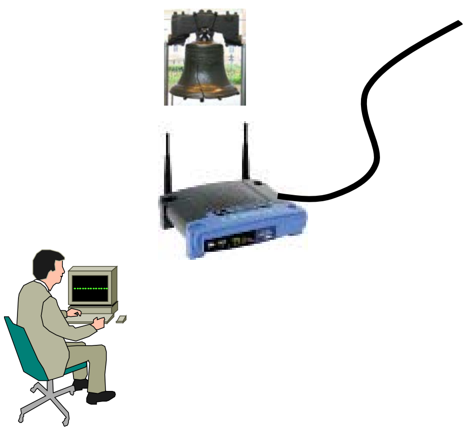
Outline

- Background and motivations
- Model formulation
- Analysis overview
- Results and their interpretation
- Extensions

What Is “User-Provided Connectivity”?

- In a UPC system, some users allow other users to access their own connectivity
  - FON, Keywifi, Community-based networks
- Different compensation schemes
  - Reciprocation, payments, or cost sharing
- An organic growth model
  - Service value depends on (grows?) with its user-base
- A simple example

Internet



- Bob connects to the Internet in Philadelphia
- Jane connects to the Internet in Paris

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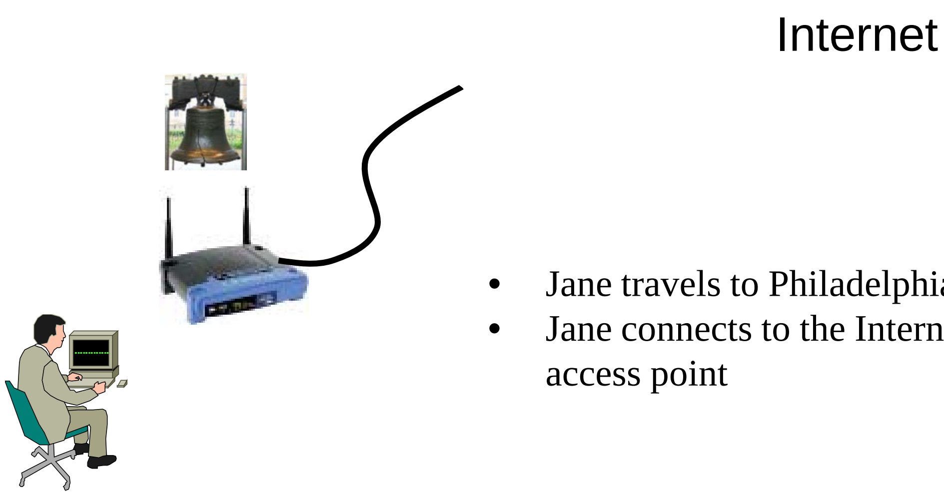

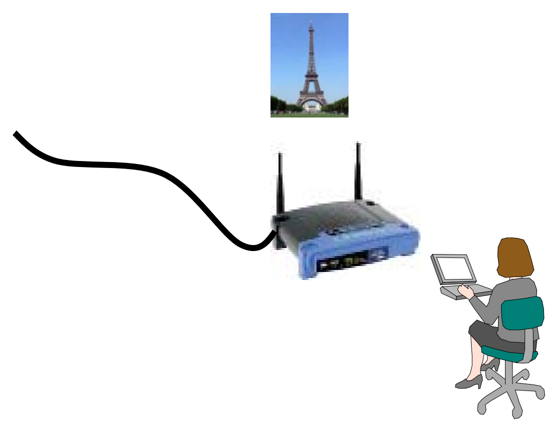

- Jane travels to Philadelphia
- Jane connects to the Internet through Bob’s access point

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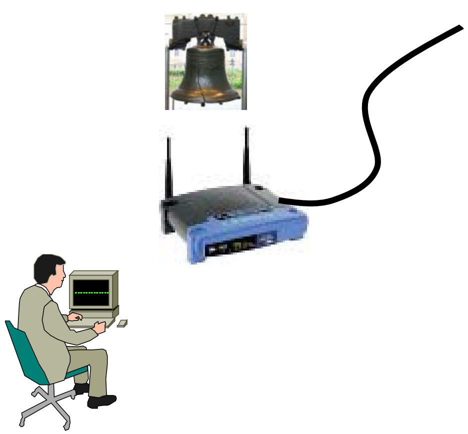


- Bob travels to Paris
- Bob connects to the Internet through Jane’s access point

Why "User-Provided Connectivity"?

- We want connectivity everywhere, all the time
  - Wireless technology has brought us very close to truly ubiquitous physical connectivity
- But, traditional "infrastructure" solutions, *e.g.*, cellular, have a high up-front cost as well as capacity limitations
- UPC offers an alternative, organic growth option
  - Limited up-front cost, and increasing capacity as more users join
  - But UPC systems face "bootstrapping" issues (positive and negative externalities)

Q: When and how can a UPC solution succeed?

Modeling UPC Adoption (Or Not)

- A model should allow us to characterize adoption outcomes as a function of exogenous parameters and users behavior
- Among a large user population, individual users adopt if the *utility* they derive from the UPC service is non-negative
- A user's utility should capture
  - Value of basic (home) connectivity
  - Value of connectivity while "roaming"
  - Impact of roaming traffic on home connectivity
  - Incentives for accommodating roaming traffic
  - Service price

  (Involve externalities)

and allow *heterogeneity* in how users value the service

A General Users’ Utility

- Users are heterogeneous in their propensity to seek connectivity “away from home,” *i.e.*, their roaming profile
  - Captured by a roaming random variable θ
    - Known distribution
    - $\theta \in [0,1]$, $\theta = 0$ (never roams), $\theta = 1$ (always roaming)
- Utility function of user with roaming value θ

$$U(\theta) = F(\theta,x)+G(\theta,m) - p_\theta$$

  - $F(.,.)$ is utility of connectivity (at home and while roaming)
    - x is current level of adoption (coverage assumed function of adoption)
  - $G(.,.)$ accounts for negative impact of roaming traffic, and positive impact of possible compensation
    - m is current volume of roaming traffic (depends on number **and** identity –their θ values– of adopters)
  - p_θ is price charged to user with roaming value θ
- A user adopts if $U(\theta) \geq 0$

A Simple Instantiation

- Linear (positive and negative) externalities

$$U(\theta) = (1 - \theta)\gamma + d\theta x + (b - c)m - p$$

- θ : Uniformly distributed in [0,1]
- γ : Value of home connectivity (affected by a user's roaming frequency)
- d : Value of connectivity while roaming $d > \gamma$ (affected by both coverage x, and a user's roaming frequency θ)
- b : Compensation for providing access to roaming traffic (proportional to volume of roaming traffic m)
- c : Impact of volume of roaming traffic, m, on user connectivity
- m : Roaming traffic uniformly distributed across users' home connections
- p : Service price (identical for all users)

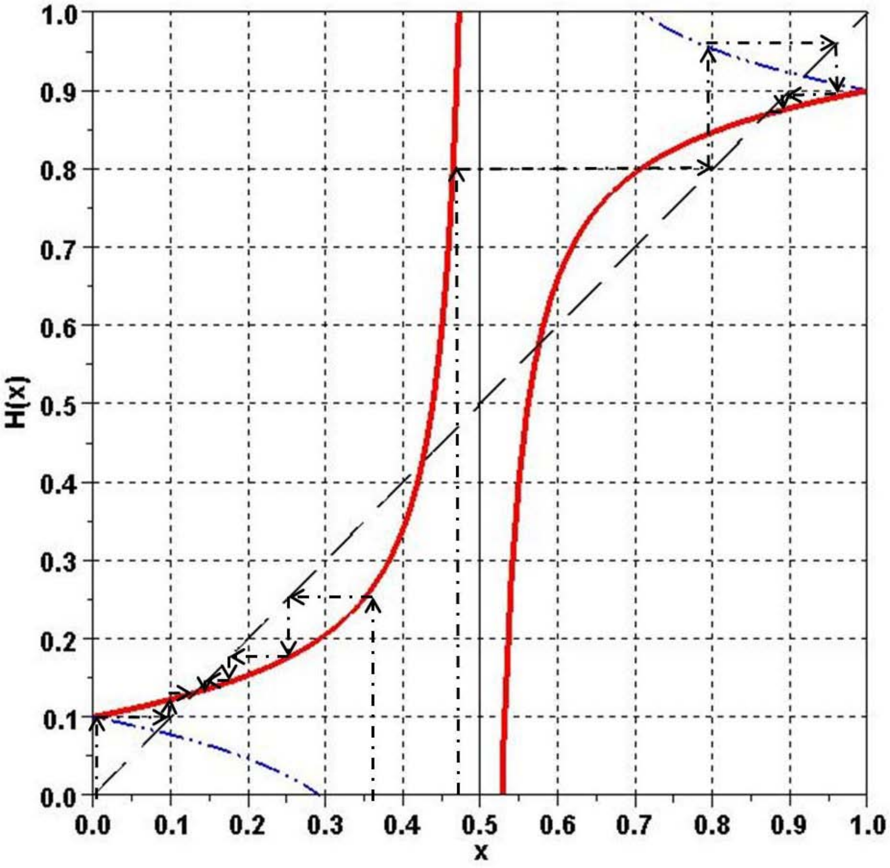
- Equivalent formulation: $U(\theta) = (\gamma - p) + (b - c)m + \theta(dx - \gamma)$

  In the paper we use $d \sim 2\gamma$, so that

$$U(\theta) = k + lm + \theta(2x - 1), \text{ where } k = (\gamma - p)/\gamma \text{ and } l = (b - c)/\gamma$$

Adoption Model & Evolution

- A simple discrete time model
  - Adoption level at epoch n+1, x_{n+1}, is determined by adoption *state* at epoch n, X_n (a two-dimensional quantity – number x_n ***and type*** y of adopters)
  - users evaluate their utility based on X_n and adopt if it is non-negative, *i.e.*, $X_{n+1}=H(X_n)$
- Adoption evolves based on the shape and position of the function(s) $H(X)$ relative to X
  - Different functions before and after a transition to a state of high/low adoption
- Equilibria correspond to $H(X) = X$ (or $H(0)\leq 0$, or $H(0)\geq 1$)

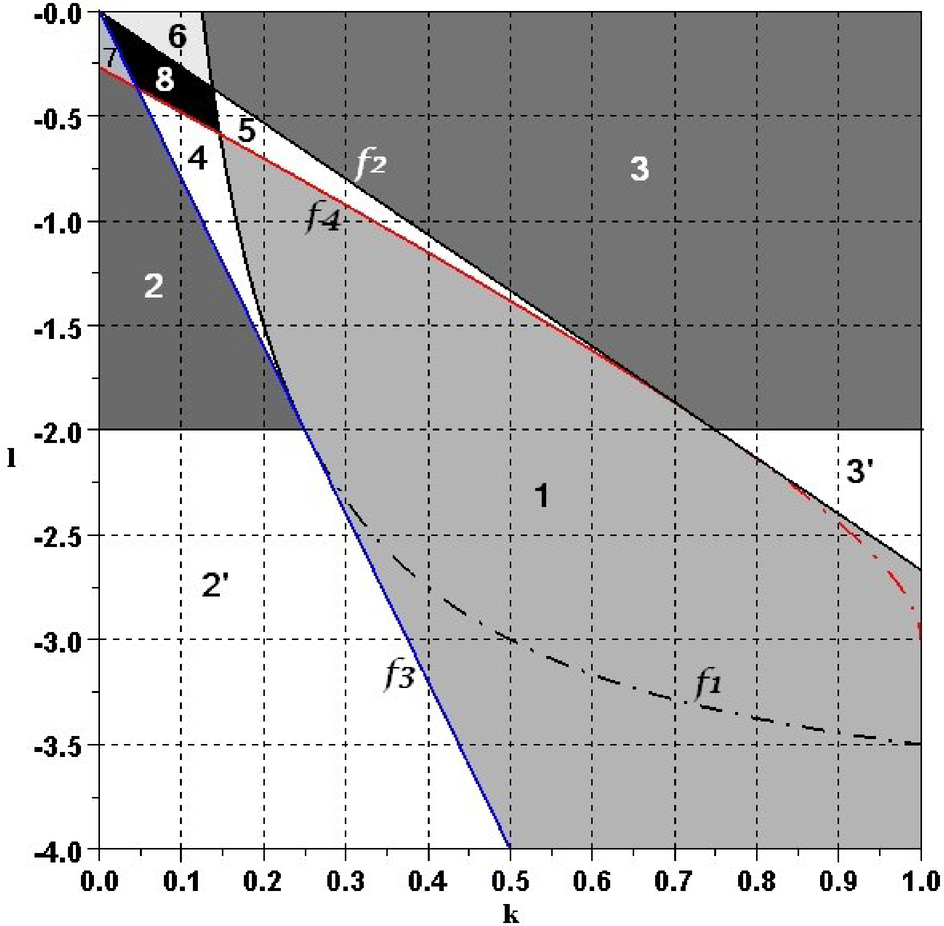


Adoption Outcomes

- Associated with different regions of the (k,l) plane
  - $U(\theta) = k + lm + \theta(2x - 1)$
- Various possible combinations of equilibria or absence thereof

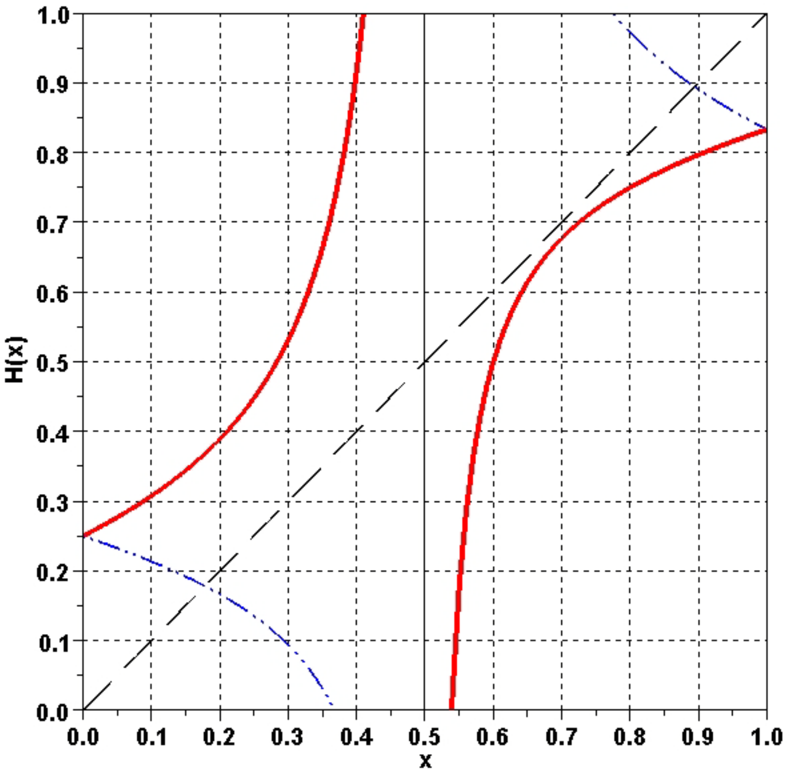
Cases	$[0, 1/2)$	$[1/2, 1]$
1	—	—
2	●	—
2'	↻	—
3	—	●
3'	—	↻
4	●, ○	—
5	—	●, ○
6	●, ○	●
7	●	●, ○
8	●, ○	●, ○

Recall that $k = (\gamma - p)/\gamma$ and $l = (b - c)/\gamma$

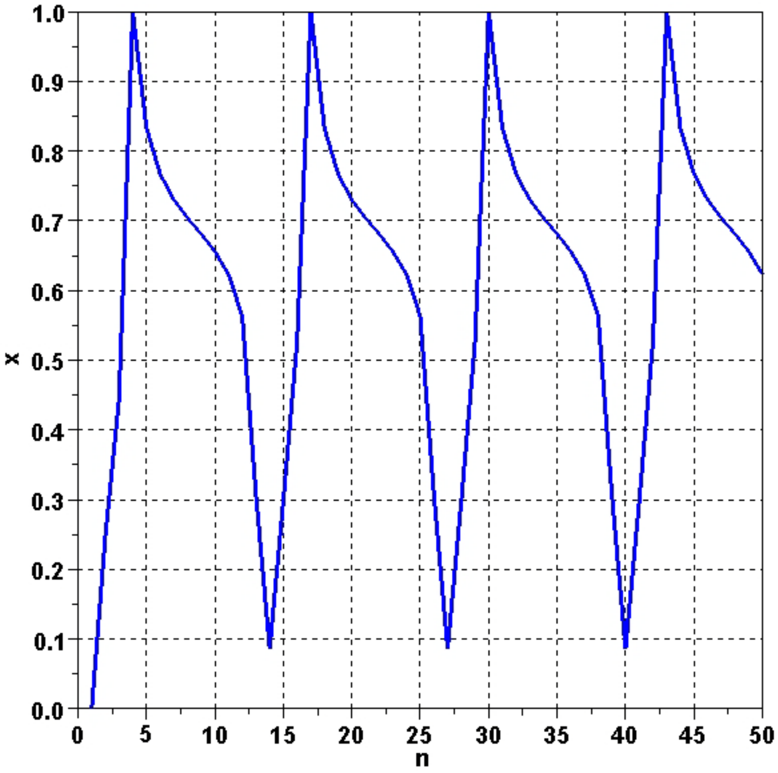


Representative Outcomes (1)
Absence of Equilibria

H(*X*) functions

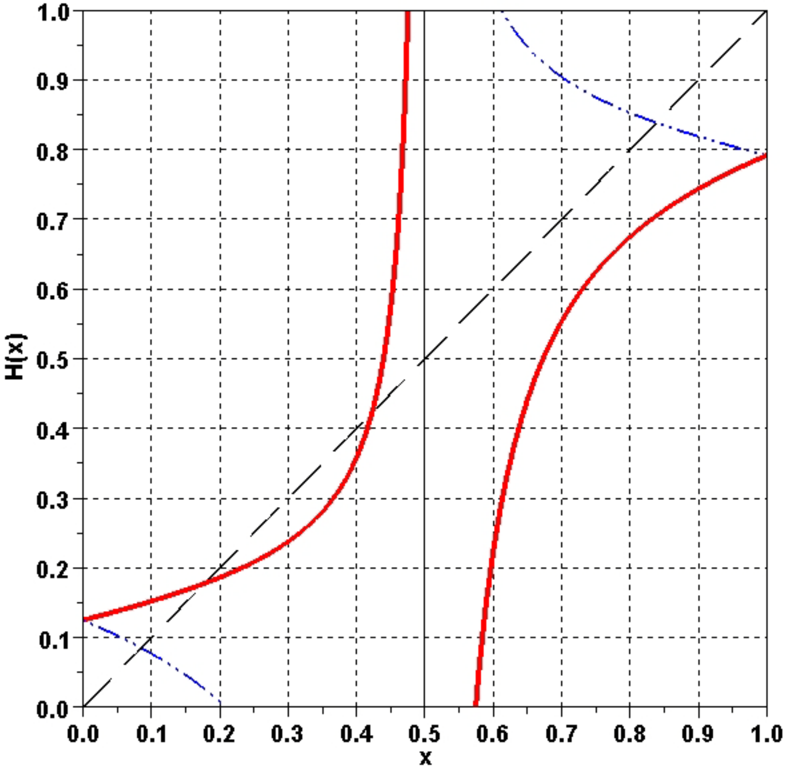


Adoption Evolution

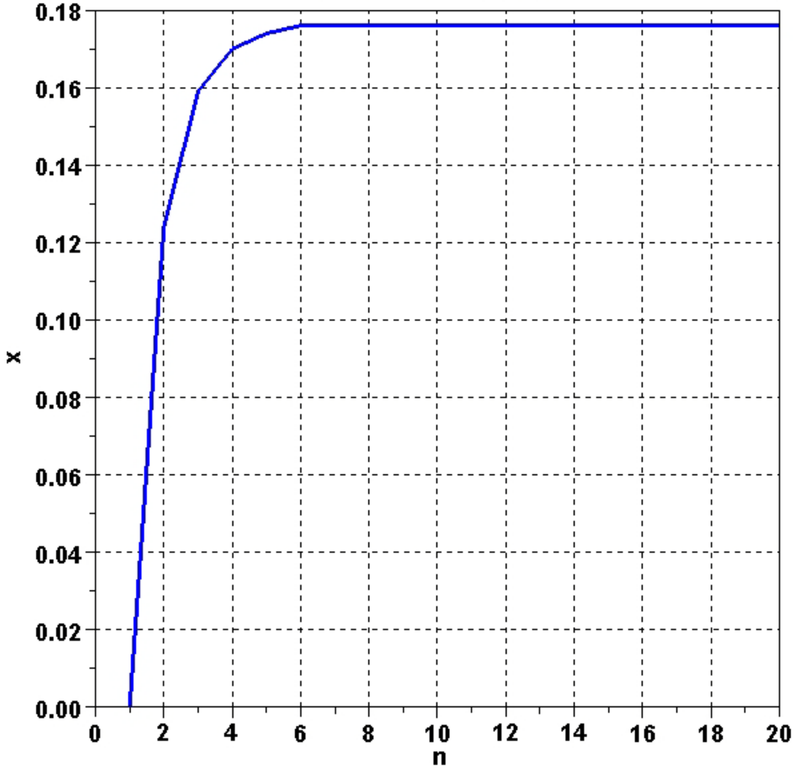


Representative Outcomes (2)
Single Stable Equilibrium (Low Adoption)

H(*X*) functions

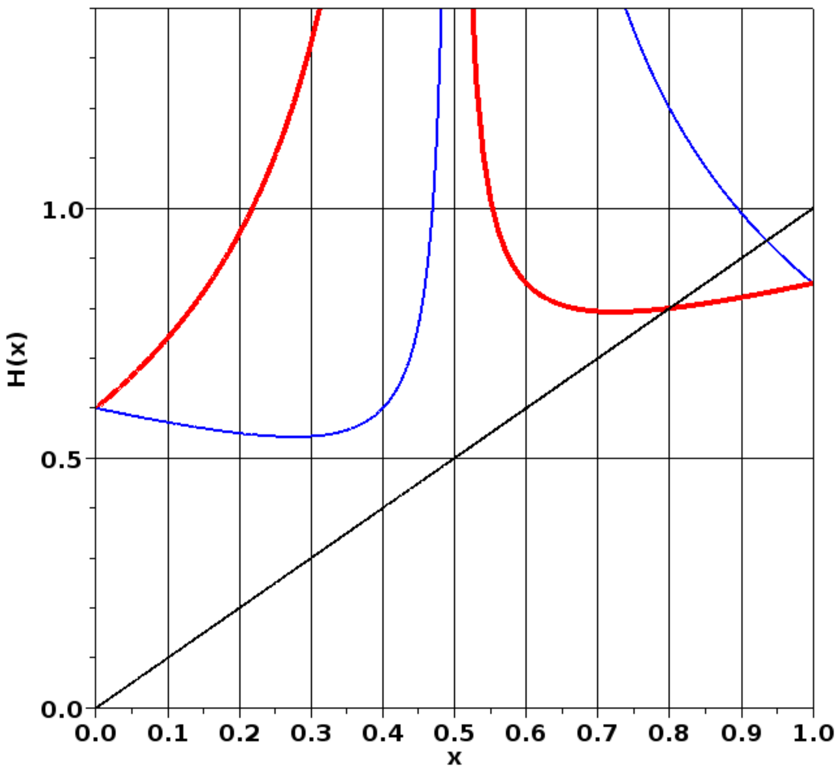


Adoption Evolution

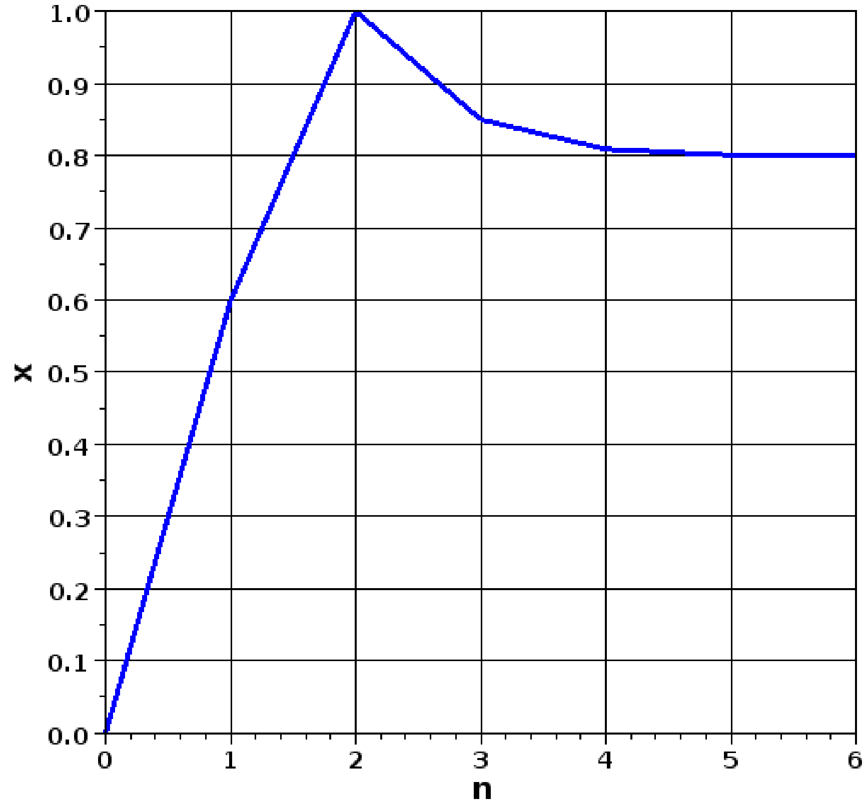


Representative Outcomes (3)
Single Stable Equilibrium (High Adoption)

H(*X*) functions

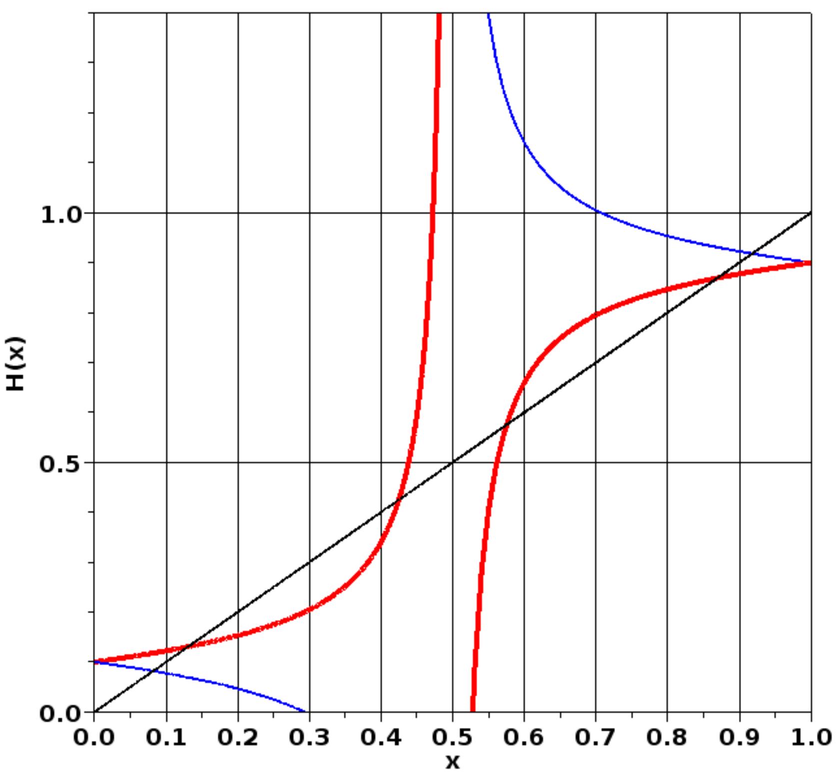


Adoption Evolution

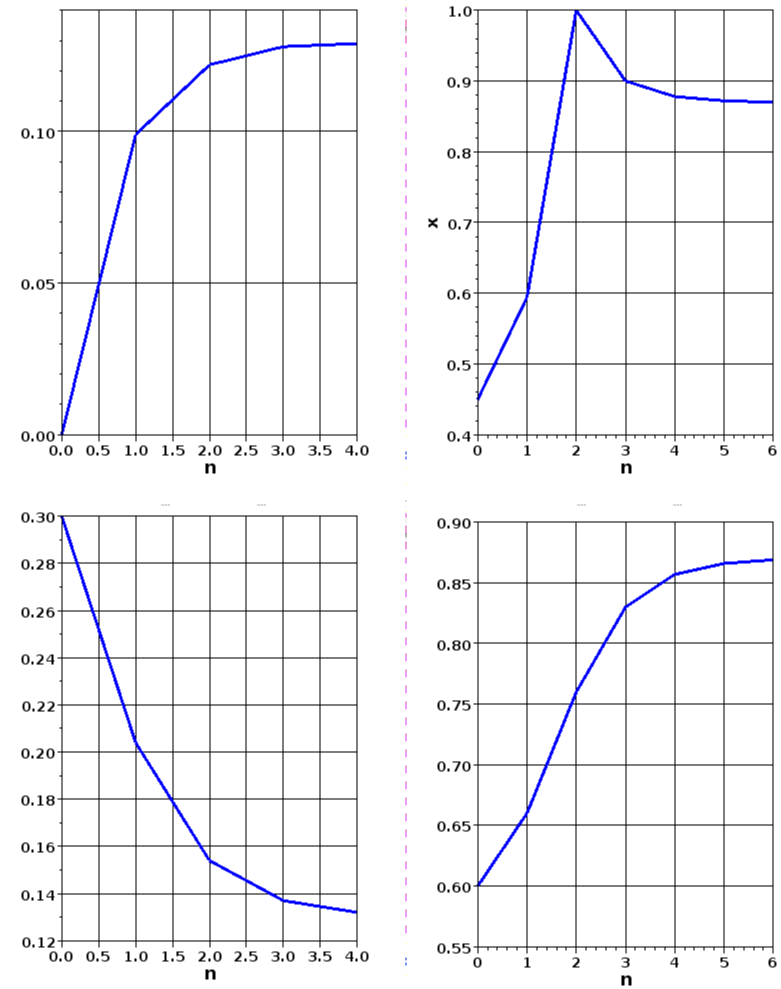


Representative Outcomes (4)
Two Stable Equilibria (High & Low Adoption)

H(*X*) functions



Adoption Evolution(s)

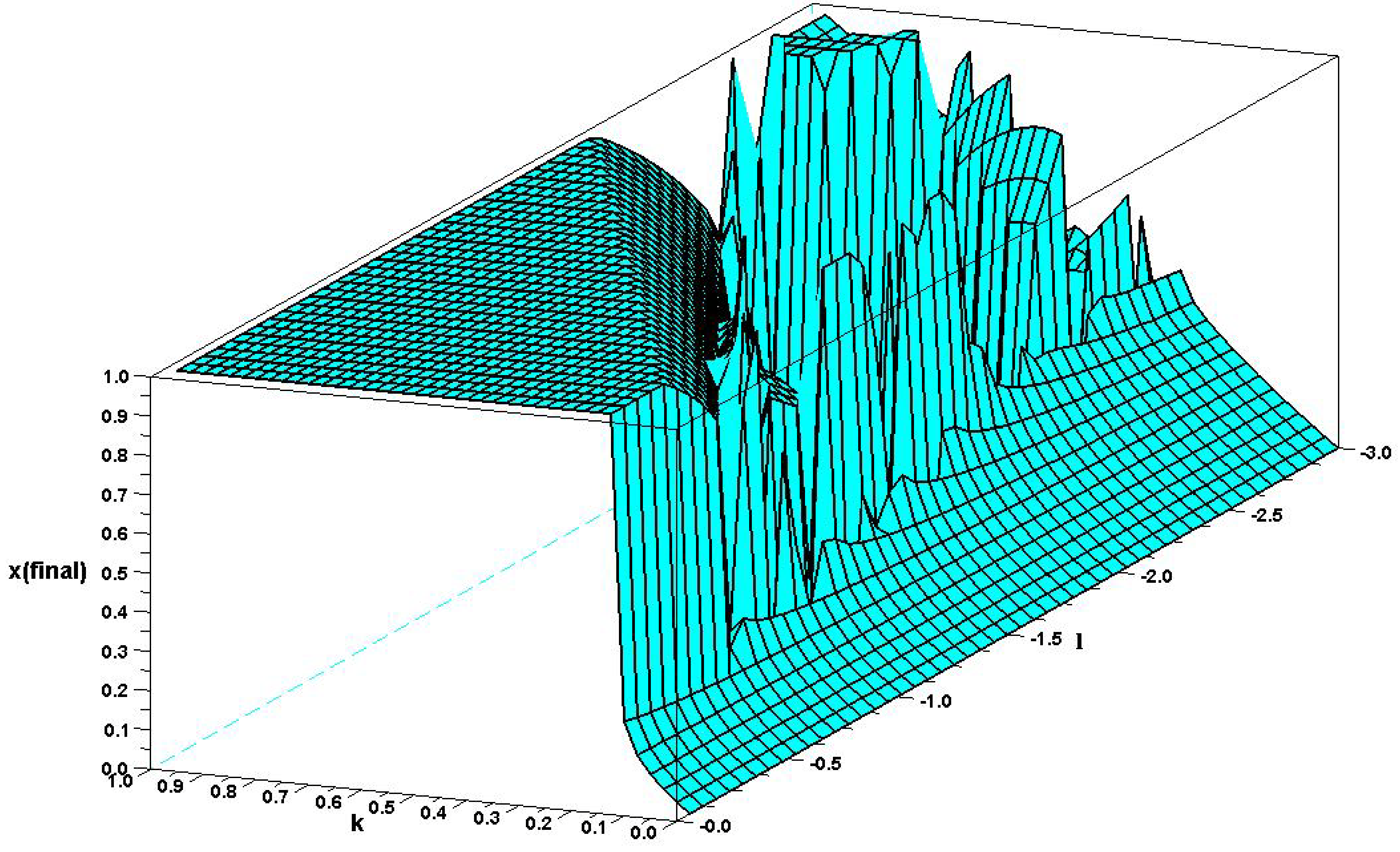


Summarizing Outcomes

- Three main “types” of behaviors

1. No or unlikely convergence to a stable adoption equilibrium
   - Primarily in regions where roaming traffic has a strong negative impact that creates boom-and-bust cycles
2. Convergence to a global, stable equilibrium
   - Convergence can be to either low or high adoption depending on how valuable connectivity is
3. Convergence to one of two possible stable equilibria depending on initial adoption (seeding)
   - Impact of roaming traffic is low, but so is value of connectivity
     - Without enough of an initial critical mass, adoption never takes off
     - Large enough seeding helps overcome initial inertia

An Evolution Snapshot

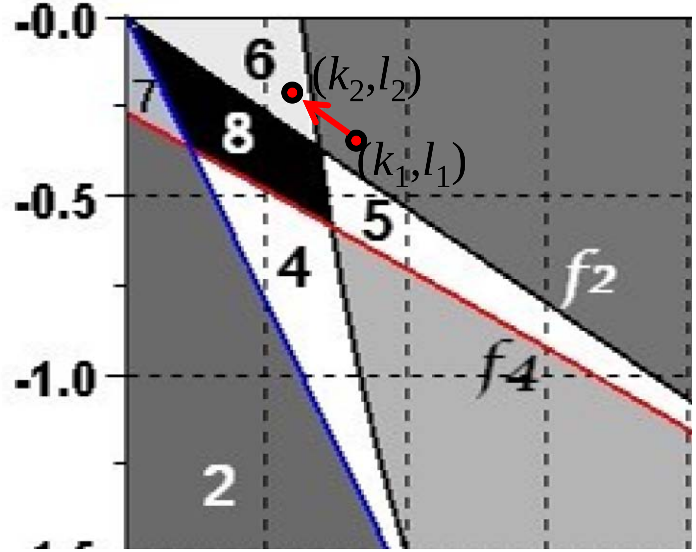


On the Impact of Incentives

- Recall a user's utility function
  - $U(\theta) = k + lm + \theta(2x - 1)$, for $\gamma=1$, where $k= 1 - p$ and $l=b - c$, with b corresponding to incentives to offset the impact of roaming traffic
- At equilibrium bm is equivalent to a decrease in price, *i.e.*, $p' = p - bm$, but impact on adoption dynamics can be significant, *i.e.*, because of the possible introduction of a ***second*** low adoption equilibrium

$(k_1,l_1) = (1-p_1, -c)$, *i.e.*, $b=0$

$(k_2,l_2) = (1-(p_1+\boldsymbol{bm}), \boldsymbol{b}-c)$



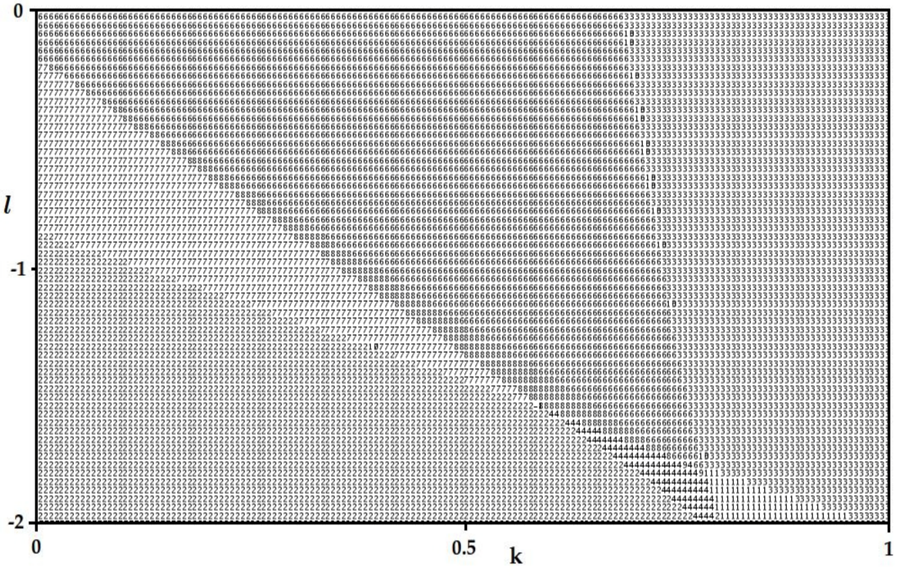
- In general, UPC adoption can be difficult to predict in the presence of multiple equilibria

Results Robustness

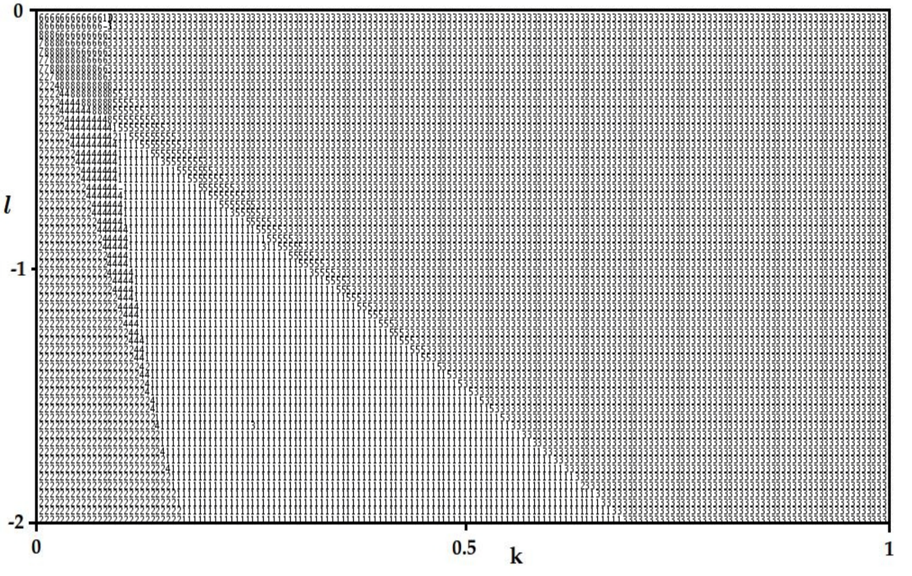
- Both users' utility and adoption processes are overly simplified, even if they capture key aspects
- Are the results affected when relaxing the model's assumptions?
  - Non-linear externality functions
  - Non-uniform roaming characteristics
  - A more realistic (diffusion-like) adoption process
- Assessing behavior of relaxed models must be done numerically
- Adoption trajectories can differ and region boundaries shift, but general outcomes remain unchanged

Shifting Regions Boundaries

Unimodal roaming (θ=1) and non-linear externalities

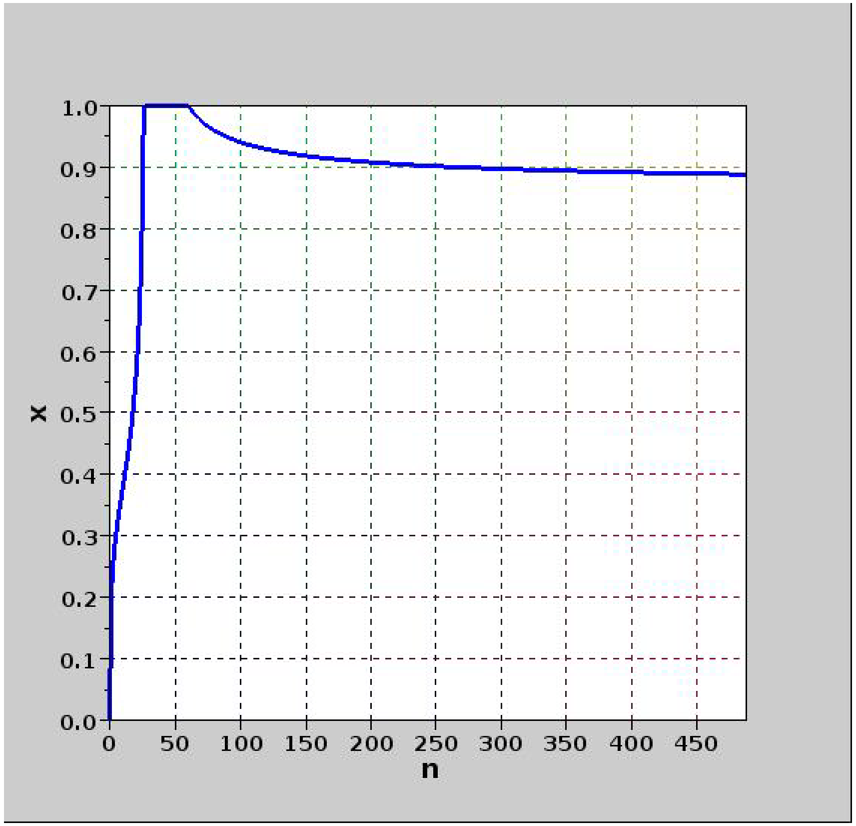


Unimodal roaming (θ=0) and linear externalities

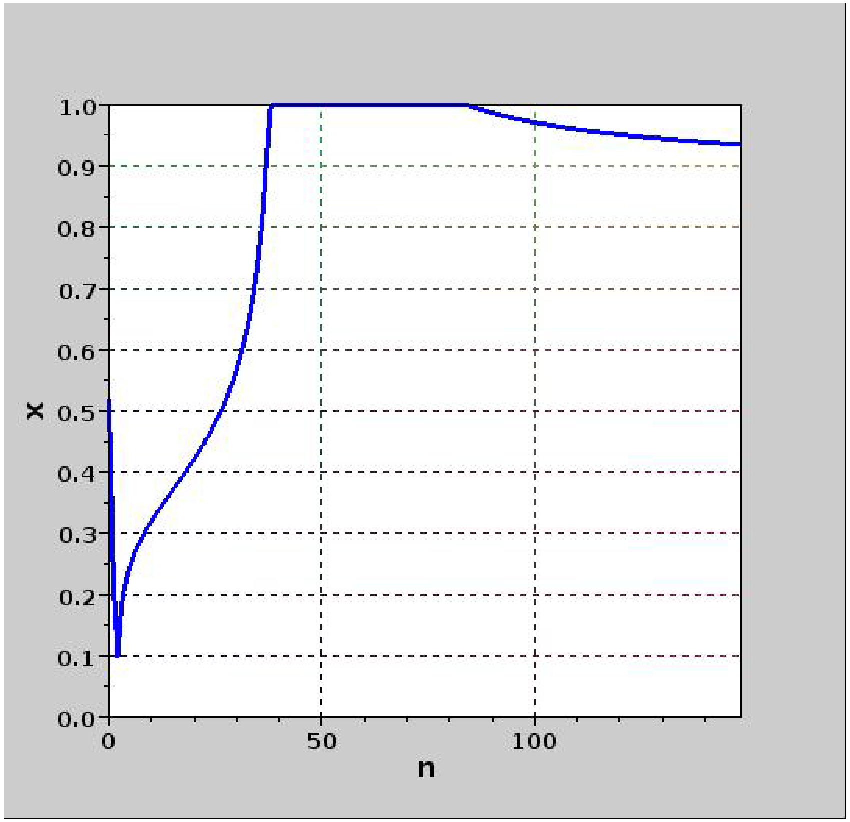


A Diffusion-like Adoption Model
Two levels of initial penetration (seeding)

No seeding



>50% initial seeding



Summary and Next Steps

- Basic model helps identify range of outcomes for UPC adoption, and impact of exogenous parameters
  - High sensitivity to roaming traffic can disrupt adoption even when connectivity is highly desirable
  - Unless value of connectivity is high, adoption may never gather enough of a critical mass for the service to succeed (potential impact of incentives)
- Obvious next step is to use the results to understand how to best set service prices
- Additionally, the results point to the limitations of a single price policy
  - Must be low enough to foster initial adoption, which fails to extract the higher final service value when adoption is high
  - A natural "fix" is to use a two-tier pricing, *i.e.*, introductory pricing followed by higher pricing when adoption becomes high enough
    - How do we set both prices and when do we switch to the higher price?